Star Formation

Q & A Session 23.06.2020

Massive Star Formation

Quiz

1) Massive Star Formation

Which of the following statements about massive star formation is true? (multiple selections possible)

1. Massive Stars are rare.
2. The closest site of massive star formation is many thousand parsec away.
3. Massive stars can form from any molecular cloud.
4. Massive stars form only from very dense molecular clouds.
5. The majority of newly formed stars has masses above 5 $M_\odot$.
6. Massive stars produce the majority of the stellar light.
7. Most of the stellar mass is bound in low-mass stars.
8. Low- and high mass stars form similarly.
9. Massive stars always form in multiple (binary, ...) systems.

Solution

1. Massive Stars are rare. - TRUE
2. The closest site of massive star formation is many thousand parsec away. - FALSE: The Orion nebula is an active site of massive star formation and its distance is 413 pc.
3. Massive stars can form from any molecular cloud. - FALSE
4. Massive stars form only from very dense molecular clouds. - TRUE
5. The majority of newly formed stars has masses above 5 $M_\odot$. - FALSE: Given the observed IMF, the number of stars is dominated by the low masses. -
6. Massive stars produce the majority of the stellar light. - TRUE: Given the observed IMF, the luminosity is strongly dominated by the high masses.

7. Most of the stellar mass is bound in low-mass stars. - TRUE: the lower masses dominate the total mass in a stellar population.
8. Low- and high mass stars form similarly. - FALSE (see lecture)
9. Massive stars always form in multiple (binary, ...) systems. - FALSE: Most of the massive stars occur in multiple systems but not all.

2) Observations of massive star formation

Observing massive star formation is complicated or assisted by (select the TRUE facts):

1. Massive star formation only takes place in the Milky Way plane and therefore suffers from strong dust attenuation.
2. Massive star formation only happens in the Milky Way halo and is therefore easy to observe.
3. Massive star formation only takes place in star burst galaxies which are very distant.
4. Massive star formation is much faster than regular star formation which makes it difficult statistically to observe active star formation.
5. Massive stars form and enter the ZAMS while they are still accreting and are still embedded in the dusty envelope.
6. Strong stellar winds during the massive star formation blur observations and confuse the detectors.
7. Massive star formation in molecular clumps happens deeply embedded with high magnitudes of visual extinction A_V.
8. Massive star formation in molecular clumps has no preferred spatial distribution and can happen everywhere.
9. Massive star formation and emission of ionized Hydrogen (HII regions) are always observed together.

Worked Example: HII Region Trapping

Consider a star of radius R_* and mass M_* with ionizing luminosity S photons s^{-1} at the center of a molecular cloud. For the purposes of this problem, assume that the ionized gas has constant sound speed $c_i = 10\,\text{km}\,s^{-1}$ and case B recombination coefficient $\alpha_B = 2.6 \times 10^{-13}\,\text{cm}^{-3}\,s^{-1}$.

a)

Suppose the cloud is accreting onto the star at a constant rate $\dot{M}_*$. The incoming gas arrives at the free-fall velocity, and the accretion flow is spherical. Compute the equilibrium radius r_i of the ionized region, and show that there is a critical value of $\dot{M}_*$ below which $r_i >> R_*$. Estimate this value numerically for $M_* = 30\,M_\odot$ and $S = 10^{49}\,s^{-1}$. How does this compare to typical accretion rates for massive stars?

The density profile of the accretion flow is given implicitly by

$$\dot{M}_* = 4\,\pi\,r^2\,\rho\,v_{\text{ff}} \tag{1}$$

where $v_{ff} = \sqrt{2\,G\,M_*/r}$. Thus we have

$$\rho = \frac{\dot{M}_*}{4\,\pi\,\sqrt{2\,G\,M_*}}\,r^{-3/2} \tag{2}$$

Recombinations happen in the region between R_* and r_i. The recombination rate per unit volume is $\alpha_B\,n_e\,n_p = 1.1\,\alpha_B(\rho/\mu_H)^2$, where $\mu_H = 2.3\times10^{-24}\,g$ g is the mass per H nucleus assuming standard composition. Thus the total recombination rate within the ionized volume is

$$\Gamma = \int_{R_*}^{r_i} 4\,\pi\,r^2(1.1\,\alpha_B)\left(\frac{\dot{M}_*}{4\,\pi\,\sqrt{2\,G\,M_*}}\,r^{-3/2}\right)^2 dr = \frac{1.1\,\alpha_B\,\dot{M}_*^{\,2}}{8\,\pi\,\mu_H^2\,G\,M_*}\ln\left(\frac{r_i}{R_*}\right) \tag{3}$$

Since this must equal the ionizing photon production rate ($\Gamma = S$), we can solve for r_i:

$$r_i = R_*\exp\left(\frac{8\,\pi\,\mu_H^2\,G\,M_*}{1.1\,\alpha_B\,\dot{M}_*^{\,2}}\right) \tag{4}$$

The condition that $r_i >> R_*$ is satisfied if the term inside parentheses is $\gtrsim 1$, which in turn requires

$$\dot{M}_* \lesssim \left(\frac{8\,\pi\,\mu_H^2\,G\,M_*\;S}{1.1\,\alpha_B}\right)^{1/2} \tag{5}$$

Plugging in the given values $M_* = 30\,M_\odot$ and $S = 10^{49}\,s^{-1}$,we obtain

$$\left(\frac{8\,\pi\,(2.3\times10^{-24})^2\,6.67\times10^{-8}\times30*2\times10^{33}\times10^{49}}{1.1\times2.6\times10^{-13}}\right)^{1/2} = 4.313236696179393\text{`}*\text{^}21\,g\,s^{-1}$$

In[∘]:= $\frac{4.313236696179393\text{`}*\text{^}21}{2\times10^{33}}*\pi\,10^7$

Out[∘]= 0.0000677522

$\dot{M}_* \lesssim 0.7\times10^{-5}\,M_\odot\,yr^{-1}$ ($\pi\,10^7$ is about $s\,yr^{-1}$). This is lower (though not by a huge amount) than the typical accretion rates inferred for massive stars.

b)

The H ii region will remain trapped by the accretion flow as long as the ionized gas sound speed is less than the escape velocity at the edge of the ionized region. What accretion rate is required to guarantee this? Again, estimate this numerically for the values given above.

The escape velocity at a distance r from the star is $v_{esc} = \sqrt{2\,G\,M_*/r}$. Thus the condition that $v_{esc} < c_i$ at r_i implies that

$$\frac{2\,G\,M_*}{c_i^2} < r_i = R_*\exp\left(\frac{8\,\pi\,\mu_H^2\,G\,M_*}{1.1\,\alpha_B\,\dot{M}_*^{\,2}}\right) \tag{6}$$

Solving for $\dot{M}_*$, we find

$$\dot{M}_* > \left[\frac{8\,\pi\,\mu_H^2\,G\,M_*\,S}{2.2\,\alpha_B\ln(v_{esc,*}/c_i)}\right]^{1/2} \tag{7}$$

where $v_{esc} = \sqrt{2\,G\,M_*/R_*}$ is the escape speed from the stellar surface. Using $R_* = 7.7\,R_\odot$ (the radius of a 30 $M_\odot$ ZAMS star) and plugging in the other input values gives

assuming $c_i = 10^6\,cm/s = 10\,km/s$

In[∘]:= $\text{vesc} = \sqrt{2 \times 6.67 \times 10^{-8} \times 30 * 2 \times 10^{33} / (7.7 \times 6.96 \times 10^{10})}$

Out[∘]= 1.22209×10^{8}

In[∘]:= $\sqrt{\frac{8\pi\,(2.3 \times 10^{-24})^2\, 6.67 \times 10^{-8} \times 30 * 2 \times 10^{33} \times 10^{49}}{2.2 \times 2.6 \times 10^{-13}\,\text{Log}\left[\frac{\text{vesc}}{10^6}\right]}}\; \frac{\pi\, 10^7}{2 \times 10^{33}}$

Out[∘]= 0.0000218539

gives $\dot{M}_* \lesssim 2.2 \times 10^{-5}\, M_\odot\, yr^{-1}$.

Problem 2: Rosseland Opacity

The aim is to calculate a mean opacity in the case of (the frequency dependent) free-free absorption in pure hydrogen.

The Rosseland mean opacity $\frac{1}{\kappa}$ is a weighted mean opacity with the weighting $\frac{\partial B_\nu}{\partial T}$:

$$\frac{1}{\kappa} = \frac{\int \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T}\, d\nu}{\int \frac{\partial B_\nu}{\partial T}\, d\nu} \tag{8}$$

Particularly, strong weight is given to those frequencies, where the radiation flux is large. The frequency-dependent opacity is given by the expression:

$$\kappa_\nu\, \rho = 1.32 \times 10^{56}\, \frac{\rho^2\, g_{ff}}{\nu^3\, T^{1/2}} \left(1 - e^{-h\nu/kT}\right) \text{cm}^{-1} \tag{9}$$

where g_{ff} is a constant quantum mechanical correction factor called the Gaunt factor.

a) $\partial B_\nu / \partial T$

The Planck function is

$$B_\nu = \frac{2 h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \tag{10}$$

First derive an expression for $\partial B_\nu / \partial T$.

Solution

$$B_\nu = \frac{2 h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \tag{11}$$

$B[\nu\_] := \frac{2 h \nu^3}{c^2} \frac{1}{e^{h\nu/(k T)} - 1}$

$\text{Dt}\left[\frac{2 h \nu^3}{c^2} \frac{1}{e^{h\nu/(k T)} - 1}, T, \text{Constants} \to \{h, c, k, \nu\}\right]$ // Simplify

$\frac{2\, e^{\frac{h\nu}{kT}}\, h^2\, \nu^4}{c^2 \left(-1 + e^{\frac{h\nu}{kT}}\right)^2 k\, T^2}$

b)

Next, introduce a dimensionless variable $x = h\,\nu / k\,T$ and eliminate ν from $\partial B_\nu / \partial T$.

Solution

```
% /. {h ν/(k T) → x}
```

$$\frac{2\,e^x\,h^2\,\nu^4}{c^2\,(-1+e^x)^2\,k\,T^2}$$

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% /. {h^2 ν^4/(k T^2) → x^4 T^2 k^3/h^2}
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$$\frac{2\,e^x\,k^3\,T^2\,x^4}{c^2\,(-1+e^x)^2\,h^2}$$

c)

Starting from equation (9), derive an expression for $\frac{1}{\rho\,\kappa_\nu}\partial B_\nu / \partial T$ and plot the resulting function. Use the plot to argue that the Rosseland mean opacity is largely determined by κ_ν when the frequency is a few times $k\,T / h$.

Solution

$$\kappa_\nu\,\rho = 1.32\times 10^{56}\,\frac{\rho^2\,g_{ff}}{\nu^3\,T^{1/2}}\left(1-e^{-h\,\nu/k\,T}\right)\text{cm}^{-1} \tag{12}$$

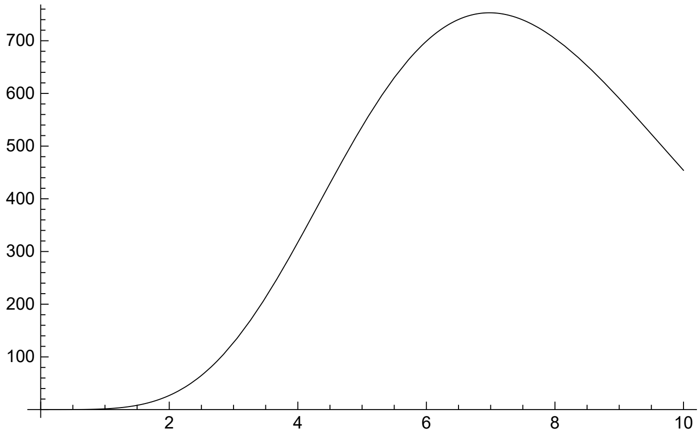
$$\kappa_\nu\,\rho = 1.32\times 10^{56}\,\frac{\rho^2\,h^3\,g_{ff}}{k^3\,T^{7/2}}\,\frac{(1-e^{-x})}{x^3}\,\text{cm}^{-1} \tag{13}$$

$$\frac{1}{\kappa_\nu\,\rho}\,\frac{\partial B_\nu}{\partial T} = \frac{1}{1.32\times 10^{56}\,\frac{\rho^2\,h^3\,g_{ff}}{k^3\,T^{7/2}}\,\frac{(1-e^{-x})}{x^3}}\,\frac{2\,k^3\,T^2}{h^2\,c^2}\,\frac{x^4\,e^x}{(e^x-1)^2} \tag{14}$$

$$\frac{1}{\kappa_\nu\,\rho}\,\frac{\partial B_\nu}{\partial T} = \frac{2\,k^6\,T^{11/2}}{1.32\times 10^{56}\,\rho^2\,h^5\,c^2\,g_{ff}}\,\frac{x^7\,e^x}{(e^x-1)^2\,(1-e^{-x})} \tag{15}$$

$$\frac{1}{\kappa_\nu\,\rho}\,\frac{\partial B_\nu}{\partial T} = \frac{2\,k^6\,T^{11/2}}{1.32\times 10^{56}\,\rho^2\,h^5\,c^2\,g_{ff}}\,\frac{x^7\,e^{2\,x}}{(e^x-1)^3} \tag{16}$$

$$\text{Plot}\left[\frac{x^7 e^{2x}}{(e^x-1)^3}, \{x, 0, 10\}\right]$$



We see that the largest contribution comes from x~7, or ν~7 $\frac{kT}{h}$.

d)

Show that the Rosseland mean opacity (equation (8)) for free-free absorption obeys Kramers law where $\kappa \propto \rho\, T^{-3.5}$.

Solution

$$\frac{1}{\kappa} = \frac{\int \frac{1}{\kappa_\nu}\frac{\partial B_\nu}{\partial T}\, d\nu}{\int \frac{\partial B_\nu}{\partial T}\, d\nu} \tag{17}$$

$$\propto \frac{T^{11/2}\rho^{-1}}{T^2}\,\frac{\int \frac{x^7 e^{2x}}{(e^x-1)^3}\, d\nu}{\int \frac{x^4 e^x}{(e^x-1)^2}\, d\nu} = \frac{T^{13/2}\rho^{-1}}{T^3}\,\frac{\int \frac{x^7 e^{2x}}{(e^x-1)^3}\, d\nu}{\int \frac{x^4 e^x}{(e^x-1)^2}\, d\nu} \tag{18}$$

$$\frac{1}{\kappa} \propto \frac{T^{13/2}\rho^{-1}}{T^{6/2}} \propto T^{7/2}\rho^{-1} \tag{19}$$

Magnetic Torque in a thin disk

Derive an estimate of the torque acting on a geometrically thin accretion disc due to a dipolar magnetic field that originates in a rotating central star. Assume that the disc has an inner edge located away from the star at radius R_{min}.

The estimate of the total torque exerted on the inner part of the accretion disc by the magnetic field is

$$\mathcal{T} = \frac{4\pi}{3}\,\frac{B_z^2(R_*)}{\mu_0}\,\frac{R_*^6}{R_{min}^3} \tag{20}$$

where $B_z(R_*)$ is the component of the magnetic flux density in the direction perpendicular to the disc at the surface of the star, R_* is the radius of the star, and μ_0 is the permeability of free space.

By equating this magnetic torque to the internal viscous torque acting at the inner radius R_{min}, derive an expression for the inner radius of an accretion disc that is truncated by the stellar magnetic field.

The inner radius R_{min} and the radius of the central star R_* are related by

$$\frac{R_{min}}{R_*} = \left(\frac{4\pi B_z^2 (R_*) R_*^{5/2}}{3\sqrt{G m}\, \dot{m}\, \mu_0}\right)^{2/7} \qquad (21)$$

where $\dot{m}$ is the mass flow rate through the disc, and M is the mass of the central star. G is the constant of gravitation.

Consider a 1 $M_\odot$ T Tauri star of radius $R_* = 1\, R_\odot$, with a magnetic field strength at its surface of $B_z(R_*) = 10^{-1}$ Tesla. If the star is accreting at a rate $\dot{M} = 10^{-8}\, M_\odot\, \text{yr}^{-1}$, calculate the radius of the inner edge of the accretion disc.

$$\frac{R_{min}}{R_*} = 50 \left(\frac{B_z (R_*)}{\text{Tesla}}\right)^{4/7} \left(\frac{R_*}{R_\odot}\right)^{5/7} \left(\frac{\dot{m}}{10^{-8}\, M_\odot\, \text{yr}^{-1}}\right)^{-2/7} \left(\frac{M}{M_\odot}\right)^{-1/7} \qquad (22)$$

For the T Tauri star, putting $M = 1\, M_\odot$, $R_* = 1\, R_\odot$, $B_z(R_*) = 10^{-1}$ Tesla, $\dot{m} = 10^{-8}\, M_\odot\, \text{yr}^{-1}$, we get an inner radius of the accretion disc

$$\frac{R_{min}}{R_*} = 50 \left(10^{-1}\right)^{4/7} (1)^{5/7} (1)^{-2/7} (1)^{-1/7} = 13.413478976398627\text{`}$$

$= 13.4\, R_\odot$

One class of young stars, known as FU Orionis stars, are known to undergo outbursts in which the apparent accretion rate increases substantially above the canonical value of $\dot{M} = 10^{-8}\, M_\odot\, \text{yr}^{-1}$. If the accretion rate during outburst increases by a factor of 10^4 above this value, then calculate the radius of the inner edge of the accretion disc using the above stellar parameters. What do you think happens to the magnetic field in this case ?

$$\frac{R_{min}}{R_*} = 50 \left(10^{-1}\right)^{4/7} (1)^{5/7} \left(10^{4}\right)^{-2/7} (1)^{-1/7} = 0.9653488644416252\text{`}$$

This figure is actually smaller than the radius($R_* = 1\, R_\odot$) used for the star itself.

In this case the magnetic field lines are swept in towards the star by the increased mass flow and are essentially crushed against the stellar surface. The configuration is now very different since a boundary layer is generated.